

Musculoskeletal Simulation-based Parametric Study of Optimal Gait Frequency in Biped Locomotion

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Abstract— Researchers have hypothesized that animal locomotory patterns seen are consistent with the resonant frequencies endowed by their musculoskeletal structures. Further it is posited that systems succeed in minimizing their energy expenditure by moving at this resonant frequency. We choose to systematically study this hypothesis in the specific context of bipedal locomotion. Researchers have sought to correlate the preferred strike frequency with the resonant frequencies of the model or used indirect measurement such as oxygen consumption, electromyography (EMG) to assess expended effort. In our study, we employed virtual prototyping with a capable musculoskeletal simulation model to study the same hypothesis. We benchmark against the available literature and demonstrate that valuable insights can be obtained that can complement the current knowledge-base in biped locomotion.

I. INTRODUCTION

BIOLOGISTS who study animal locomotion have long speculated the control, structure and actuation has been optimized by evolution for locomotion. In the class of vertebrates, the focus of this paper, such locomotion is derived from cyclic/periodic motions of locomotory appendages (arms, legs, fins, wings), implemented in the form of integrated neuromusculoskeletal systems. The suitable coordination and neural control of the contraction of the muscles attached to an underlying skeletal structure allows for realization of desired periodic motion-trajectories (gaits). It is noteworthy that antagonistic co-contraction of the highly redundant musculature, has also been shown to provide significant robustness to environmental disturbances (modulated force interactions). However, our focus is on steady state gait where the consensus opinion is that they have been optimized to reduce the overall effort.

Our long-term interests are in systematically studying natural and robotic systems, from the view point of: (i) obtaining insights that will allow for better design of the robotic systems; and (ii) using the robotic counterpart as a scientific tool for studying hypotheses regarding the natural system. In particular, one commonly-discussed hypothesis is that expended effort is minimal when the locomotory appendages/propulsive structures are moving at steady-state in periodic gaits at their resonant frequencies. Ahlborn *et al.*

showed that humans, and by extension all animals, maintain resonance during walking and running [1, 2]. In this paper, we will focus on bipedal walking as a prototypical task for the following reasons: (i) It possesses considerable richness in terms of complexity of underlying neuromusculoskeletal structure and control; (ii) considerable past work (experimental and mathematical) is available to help cross-check the results; and (iii) most importantly, there is considerable interest currently in realizing bipedal robotic systems, where efficiency is extremely critical.

Various modeling, analysis and experimental approaches have been employed in the past to study this very important problem (and will be reviewed in the next section). However, in this paper, we will hypothesize that under “normal” steady-state gait conditions, the neuromusculoskeletal system has been optimized so as to minimize the expended effort when it reaches resonance. To validate this hypothesis, we propose the use of a virtual prototyping/ simulation-based-design methodology using contemporary computational tools (musculoskeletal analysis coupled with a parametric sweep/optimization tool) to facilitate the systematic study initially of simplified pendulum models and subsequently full-fledged bipedal walking, completely within a virtual environment.

The rest of this paper is organized as follows: In Section II, we discuss some of the previous work in bipedal locomotion modeling. Section III presents an overview of the test framework, which is examined and validated in the context of simple and compound pendulum case studies in Section IV. Section V focuses on the bipedal walking analysis, the results of which are discussed in Section VI. Finally, we conclude the paper with a summary and directions for future work in Section VII.

II. BACKGROUND

A. Modeling of Biped Locomotion

Alexander [3], provides a good review of early research on correlating animal and human gait speeds with their stride length. Experimental observation studies in the 1960s found that energy consumption is minimal for a particular chosen frequency and were used to establish the preferred correlation between stride frequency and walking speed. In the 1970s, various simple and elegant mathematical models were created to calculate the power expenditure during biped walking. A good introduction to the early research on biped walking is given by McMahan [4]. Perhaps the most cited model to study gait locomotion is the ‘ballistic model’

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of Mochon and McMahon [5]. Inspired by McMahon’s work, McGeer studied the passive walking [6] (‘rimless-wheel’ model) and developed a prototype biped walking robot powered solely by gravity. From a robotics perspective, mathematical models used to study human walking ranged from simple inverted pendulum model [7], double pendulum model [5], to complex multi-body model [8], or combinations thereof [9]. Rapid increases in computational power coupled with advances in terms of simulation, modeling capabilities allow for creation and use of powerful numerical simulation tools and this will be the focus of our work here.

B. Virtual Prototyping

Virtual Prototyping (VP) [10] refers to functional simulation, quantitative performance analysis and iterative refinement of suitable products and processes in a virtual environment. In this paper, we will examine the application of this VP methodology to study bipedal walking completely in a virtual environment. Quantitative performance analysis depends critically on availability and effectiveness of the functional simulation tools which are limited by the extent of capture of the underlying physics, the modeling and analysis fidelities and ultimately computational power. Varying levels of such simulations are possible – e.g. kinematic modeling that formed the basis of a user-customized development of human-worn products [10]. Computational musculoskeletal-analysis tools such as SIMM [11] or AnyBody [12] can now allow monitoring of internal human variables, such as muscle-forces and muscle activities, that we will use this in this paper.

C. Measuring Energy Consumption

In the absence of direct measures, the amounts of energy expenditure, is correlated and estimated through EMG and oxygen consumption measures or inverse dynamics techniques. The electrical signal associated with the contraction of a muscle is a commonly-used measure of muscle activity [13, 14] and can be captured as an Electromyogram (EMG). Similarly, the localized measurement of oxygen consumption, measured by blood-gas measurement, reflects the energy consumption in that muscle. Recent studies have shown good correlations of EMG and local oxygen consumption to external load as well as to muscle forces [15]. For this study, we will use a measure called “muscle-activity” as a fraction (between 0 and 1) of maximum voluntary muscle-contraction-force. In other words, the activity at a given point in the gait cycle indicates the percentage of maximum muscle force that can be exerted. Muscle activity profiles obtained from AnyBody have been shown to correlate very well with EMG data [16]. We can further post process this “activity level” to determine (i) peak values indicative of the range of muscle force required (since all begin at activity level 0); (ii) average value over a gait cycle (used to develop metabolic energy consumption estimates).

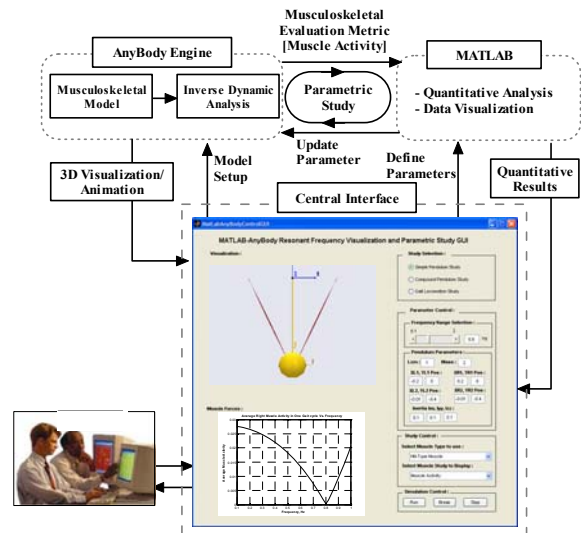


Fig. 1: The test framework that consist of a Graphical User Interface (GUI) that integrate the AnyBody musculoskeletal analysis tool and MATLAB computation engine.

III. TEST FRAMEWORK

A. Musculoskeletal System Model:

We bring the tools together in an integrated test environment as shown in Fig. 1. The framework consists of a Central Interface which interacts with musculoskeletal models. Constrained musculoskeletal system-level computational models [17] are constructed modularly using the AnyBody software package by placing physiologic and behavioral constraints on anatomical components (e.g., bone, muscle, and tendon). Alternatively, parametric models of the biped model can be extracted from libraries and potentially customized to reflect the user characteristics. A variety of quantitative and biologically relevant data such as muscle forces, joint reactions, metabolism, mechanical work, efficiency, can be easily accessed for subsequent analyses.

The parametric nature of the AnyBody software package permits the use to change many aspects of the virtual model prior to running the inverse dynamics analysis. These include the biological muscle parameters (such as types of muscles, pennation angle), effective mechanical properties (such as stiffness, strength, muscle-tendon length) and geometric properties (such as origin/ insertion points and lines of flexion of muscles). The automation of many facets of this process (using MATLAB) together with the front end (in the form of MATLAB GUI) is intended to help overcome these limitations.

B. Evaluation Metrics

As noted previously, the muscle activity profile is comparable to the EMG data, which in turn relates to muscle contraction forces and energy cost and hence is used as a metric for our study. For example, Fig. 2 shows the muscle activity profile of the ‘right muscle’ and its corresponding average muscle activity of our simple pendulum study

(shown in next section) at frequencies 0.2Hz and 0.4Hz. In our studies, we will plot the value of average muscle activity as we sweep across a range of frequencies.

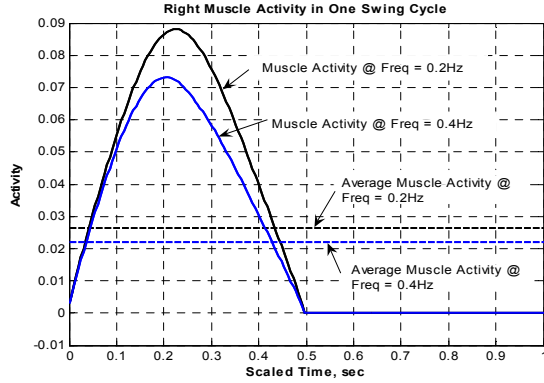


Fig. 2: Right muscle activity profile (solid) and average muscle activity (dashed) for one pendulum swing cycle.

IV. SIMPLIFIED MODELS

The simple and compound pendulum models are two commonly used simplified models in gait frequency studies [6, 18, 19]. They not only serve to validate the analytical result and simulated results, but are also provide improved understanding of the complex musculoskeletal gait model to be studied next.

A. Simple Pendulum Model

1) Analytical Formulation

The natural frequency ω of a simple pendulum with axis of rotation at a distance L_{CM} from center of mass m , is:

$$\omega = \sqrt{g / L_{CM}} / (2\pi) \quad (1)$$

where $g = 9.81m/s^2$ as the gravitational constant. Note that the natural frequency is only a function of its length.

2) Musculoskeletal Model Simulation

A simple pendulum with a concentrated mass was modeled in AnyBody, two Hill-type muscles (named ‘left muscle’ and ‘right muscle’ in Fig. 3) were created on each side of the pendulum to drive the pendulum swing at a rate of $\theta = 20^\circ \sin(\omega t)$. A complete cycle of this swing motion is shown in Fig 4. We then run the analysis with frequency ω ranging from 0.1Hz to 1.0Hz (corresponding to swing period of 10 – 1 seconds). Muscle activity profiles of right muscles for 1 gait cycle are plotted on a normalized time scale (0 – 1.0), and muscle activity profiles across frequency range 0.1Hz – 1.0Hz with step of 0.1Hz are plotted in Fig.

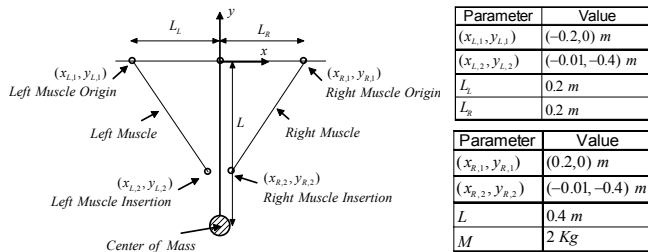


Fig. 3. The parameters setting for the simple pendulum case study.

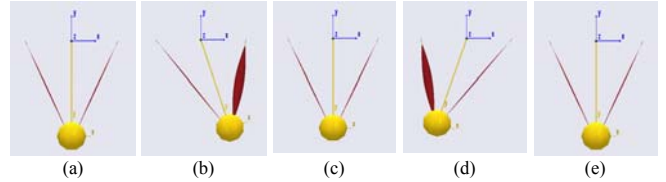


Fig. 4: (a) – (e) The complete one cycle of the simple pendulum swinging motion in study, modeled in AnyBody.

5(a) as 3D plot. As one can see, muscle activity profile decrease as the pendulum swing frequency increases, until it reaches roughly 0.8Hz, muscle activity starts to increase again. Note that after this cut-off frequency, the right muscle needs to exert force to ‘hold’ the pendulum from swinging too fast to the left – hence the increase of muscle activity. To determine this ‘cut-off’ frequency, we plot the average muscle activity (for the right muscle over one swing cycle), as well as average metabolic cost estimation across the same frequency range, with a finer increment of 0.01Hz. The results, shown in Fig. 5(b), indicate that the minimum average muscle activity occurs at frequency 0.79Hz. This result compares well with the analytical solution obtained using the simple pendulum formula given in Eq.(1), when numerical parameters are chosen as: distance from axis of rotation to center of mass $L_{CM} = 0.4m$, mass $m = 2Kg$, the resulting natural frequency ω , is 0.788Hz.

B. Compound Pendulum Model

1) Analytical Formulation

For a compound pendulum with length L and distance from axis of rotation to its center of mass (CoM) L_{CM} , mass m , inertia I , the period, and $g = 9.81m/s^2$ the natural frequency ω , is given by:

$$\omega = \sqrt{mgL_{CM} / I} / (2\pi) \quad (2)$$

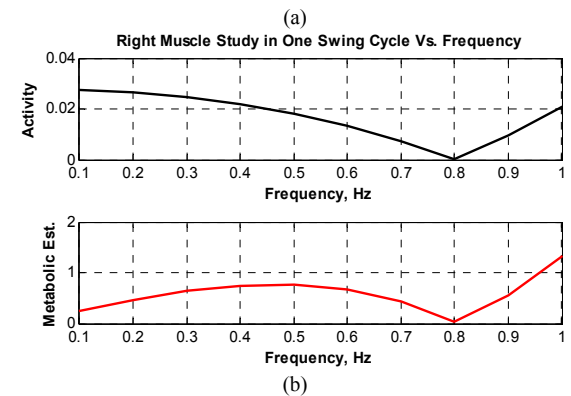
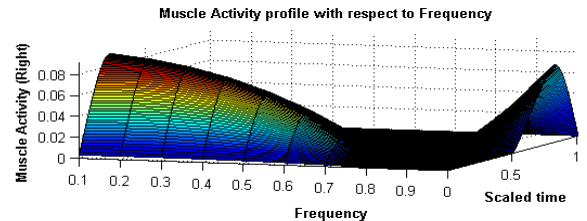


Fig. 5(a): Muscle activity profile; and (b) Average muscle activity (top) and average muscle metabolic cost estimation (bottom) of the right muscle across frequency range from 0.1 to 1.0Hz, in simple pendulum study.

A rod with axis of rotation at its end has a moment of inertia $I = \frac{1}{3}mL^2$ (where L is the total length of the rod), with center of mass located midway on the rod, $L_{CM} = L/2$, Eq. (2) can now simplify to:

$$\omega = \sqrt{3g/2L}/(2\pi) \quad (3)$$

2) Musculoskeletal Model Simulation

For ease of obtaining analytical solution, a uniform rod with length $L = 0.8m$ and center of mass at half its length is used as the compound pendulum model. The simulation setting is essentially the same as the simple pendulum case (hence will not repeat it here), except the concentrated mass of the simple pendulum is now replaced by a uniform rod, as shown in Fig. 6.

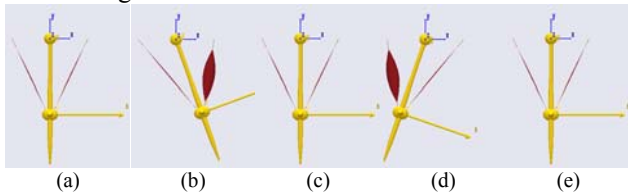


Fig. 6: (a) – (e) Complete one cycle of the compound pendulum (a uniform rod) motion in the compound pendulum study, modeled in AnyBody.

We plot the total muscle activity of a swing cycle across frequency range of 0.1Hz – 1.0Hz, in Fig. 7. We see that minimum muscle activity occurs at 0.69Hz, which compare well to the natural frequency for a uniform rod pendulum (0.6825Hz), calculated using Eq.(3).

C. Discussion of ‘Pendulum-like’ Models

Comparing Eq.(1) and Eq. (3), researchers [1] have noted that a generalized frequency expression may be written as:

$$\omega = n\sqrt{g/L}/(2\pi) \quad (4)$$

where n is a constant.

For $n = 1$, Eq. (4) become Eq. (1), is also the equation used in [18, 20] to predict the preferred stride frequency of human walking (the Force Driven Harmonic Oscillator (FDHO) model) where L_{CM} , is now the ‘simple pendulum equivalent length’ calculated based on the anthropometric data of lower extremity. Studies in both quadrupeds and human [18], shows that $n = \sqrt{2} = 1.414$ gives a better prediction of preferred stride frequency that matches with observation. If the compound pendulum is a uniform rod (the ‘stick leg’ model used in [1]), $n = \sqrt{3/2} = 1.225$; if cone is used, n would be equal to $\sqrt{5/2} = 1.581$.

V. GAIT LOCOMOTION ANALYSIS

In the previous two examples, musculoskeletal simulation showed that the minimal muscle activity coincided with motion close to the natural frequency. Using a similar approach, we will seek to predict the frequency at which minimum muscle activity occurs for bipedal walking directly from motion data collected at a particular speed. However, there are several new considerations and distinct features as compared to the pendulum model.

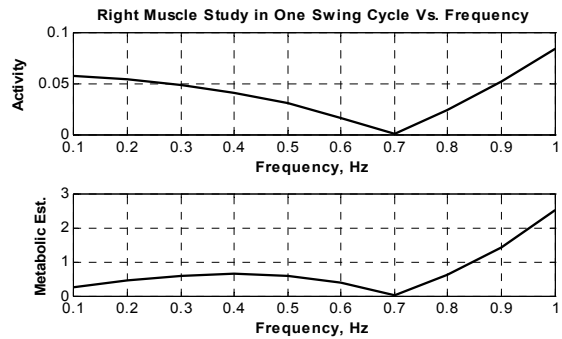


Fig. 7: Average muscle activity (top) and average muscle metabolic cost estimation (bottom), of the right muscle across frequency range from 0.1 to 1.0Hz, with increment of 0.01Hz, in the compound pendulum study.

A. The Musculoskeletal Model

The lower extremity musculoskeletal model used in this study consists of 7 rigid bodies: Thigh, Shank and Foot for both legs, and the Pelvis. Hip joints are modeled as spherical joints, knee as hinge (revolute) joint and ankle joints are considered as universal joints. This results in a model with 18 degree-of-freedom (DOF). There are 35 muscles in each leg, resulting in a total of 70 muscles in the model, and each muscle is model based on the Hill-type muscle model. Among the 35 muscles modeled in each leg, 4 of the ‘fan-shaped’ muscles that connect pelvis and femur (Gluteus Minimus, Gluteus Medius, Gluteus Maximus, and Adductor Longus) are replaced using 3 muscles to represent the triangular structure of the muscle, resulting in a total of 27 unique muscles in each leg.

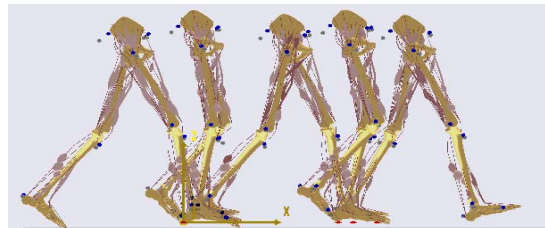


Fig. 8: A complete gait cycle from start to finish, modeled in AnyBody, used in the gait locomotion analysis.

A gait cycle is considered to begin from the point the heel touches the ground, till the heel of the same leg touches the ground again [13], as shown in Fig. 8. Motion capture data serves to provide the underlying motion-specification (gait) inputs to our model. The benchmark ‘Man’ motion capture dataset (published in [13], and downloadable from [21]) contains both motion data and the ground reaction forces. For this study, we use only the motion data which is input to the system as the kinematic trajectories of the points-of-interest corresponding to marker-locations. The original motion capture data set took 1.25 seconds to complete the gait cycle shown in Fig. 8, this corresponding to a stride frequency of 0.8Hz. The stride length of this model is about 1.24m, which result in a walking speed of 0.99m/s. This data falls within the range of a ‘Normal Man’ [13], with stride frequency 0.77Hz (Cycle time 1.3 seconds), stride length 1.28m, and an average speed of 0.99m/s. The total length of

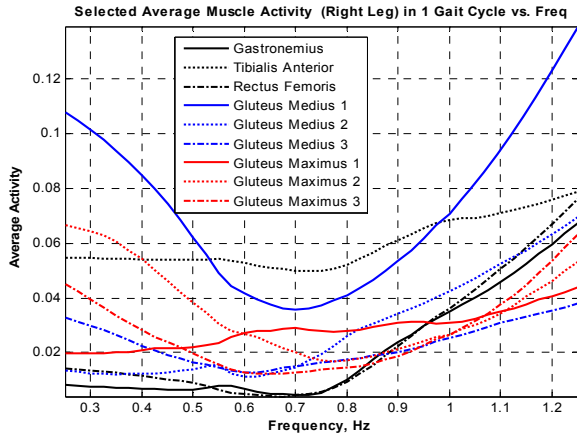


Fig. 9: Average muscle activity of selected muscles in the lower extremity from the musculoskeletal model over frequency range of 0.25Hz to 1.25Hz.

the lower extremity of this model is about 0.88m (both Thigh and Shank are 0.44m, foot is 0.22m).

B. Simulation Setting and Assumption

Both model-prediction and experimental observation literature indicates that a human can maintain ‘normal’ walking at speed up to 3m/s [22, 23], and the observed walk-run transition speed is about 1.92m/s [24]. The following assumptions were then made in our study: (i) the gait pattern remains the same for the range of speed 0.3m/s - 1.5m/s; (ii) the force and moment plate data is not considered; and (iii) the motion data set is sufficiently smooth. This gait model had been verified independently by Foster [25].

In other words, using only motion data at one speed, we seek to study what happens as a person walks at slower or faster speeds in a musculoskeletal simulation framework. We can then answer questions like: Does the preferred stride frequency of this person (0.8Hz) really consume least efforts? Is the pendulum-like setting adequate to predict the preferred walking speed?

C. Simulation Result

Parametric swept studies of a range of frequencies were carried out within AnyBody. In the subsequent analysis (in Fig.), we examine the results in the form of average muscle activity from 5 unique muscles. These five muscles (Gastrocnemius, Tibialis Anterior, Rectus Femoris, Gluteus Medius 1-3, and Gluteus Masimus 1-3) were chosen since prior studies have demonstrated good matching/correspondence between computed muscle activities with measured EMG pattern given in [13, 21].

From Fig. 9, we see that these muscles exhibit a significant trough as we sweep across the frequency range. (Most of the remaining 28 muscles exhibit similar trend but are not shown here due to space limitation). The minimum muscle activity, for most of the muscle, occurs at the range of 0.65Hz – 0.8Hz, with an averaged system-level minimum at 0.78Hz (Shown in Fig. 10).

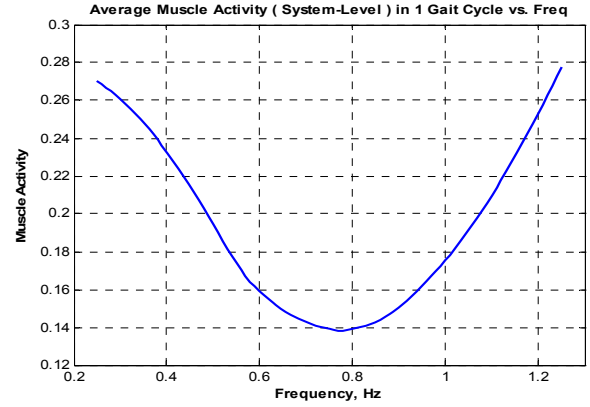


Fig. 10: System level muscle average activity plot shows that the frequency at which muscle activity is lowest located at 0.78Hz.

VI. ANALYSIS OF RESULTS

Many factors tend to make the determination of ‘real’ natural speed through observation difficult and there is considerable variability. As noted in the gait-literature, the natural speed of a human is very dependent on the environment. Average adult walking speed can range from 0.8m/s in small village to 1.7m/s in large cities [14]. Studies have also shown that variation in natural speed in long vs. short walkway, indoors vs. outdoor, walking on treadmill vs. natural surface [26].

We first compare our result with experimental study. Holt *et al* [18] study of 24 subjects (8 female and 16 male) with average leg length of 0.82m walk in a large gymnasium-size testing area, observed a mean frequency of 0.92Hz. From this perspective, we see that while our results falls within this range, the initial data set for ‘Man’ exhibits a preferred stride frequency closer to the lower end. Holt *et al* [18] also note that his FDHO model (Eq.(4) with $n = \sqrt{2}$) tends to underestimate the preferred frequency for those with higher walking frequency (>0.95 Hz) and overestimate the preferred walking frequency of those with lower walking frequency (<0.86 Hz).

We then compare our result with that of various ‘simple pendulum equivalent length’ formula given in [18, 20], the calculated preferred stride frequency of our model is 0.638Hz with $n = 1$, and 0.903Hz with $n = \sqrt{2}$. These

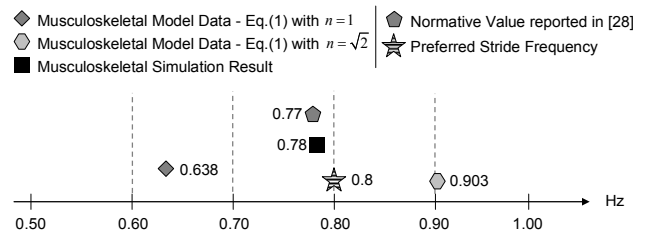


Fig. 11: Plot of frequency locations for comparison – the preferred stride frequency from the motion capture data (0.8Hz), calculated preferred stride frequency using Holt *et al.* [4] equation (Eqn. (4)) with $n = 1$ (0.638Hz) and $n = \sqrt{2}$ (0.903Hz) with anthropometric data from our model; result obtained (Fig. 11) using musculoskeletal simulation approach (0.78Hz); and normative preferred stride frequency value given in [28] (0.77Hz).

results are reflected pictorially in Fig. 11, which depict the plots of frequency locations calculated and result obtained from AnyBody simulation. We see that the preferred stride frequency (0.8Hz) compares well with the frequency in which minimum muscle activity occurs (0.78Hz) in our musculoskeletal simulation, while the predicted preferred stride frequency calculated using Eq.(4) with $n = \sqrt{2}$ over estimated the preferred stride frequency.

VII. CONCLUSION

In this study, our goal was to study the viability of musculoskeletal analysis tool for locomotion studies – in predicting the natural frequency of gait and extensions such as measuring metabolic consumption. Our first two case studies with simple pendulum and compound pendulum were intended to develop greater confidence in the use of the tool prior to deployment in a actual gait study.

The analysis of the total effort largely supports the hypothesis that animals may in-fact be choosing to minimize these metabolic consumption of energy. However, while the results show promise, we would like make some further observations which will serve to guide our future investigations. Given our initial hypothesis on resonance, we note that while a sharp resonant peak is not well defined, a region of minimal effort can be quite early delivered. The average-activity-per-gait-cycle profiles, which are closely related to total power-consumption allows for an adequate definition of a minimum to determine the operating frequencies. Further, different muscles appear to have different minima – close to each other but at adequately distinct locations. While some parts may be attributed to numerical error we believe that this may arise due to inadequately capturing the physics of the problem e.g. the treatment of large muscles and are investigating this further.

Finally, we note that in our study, we removed the ground reaction force and moment data in our study, making the leg swing like a pendulum. Masami *et al.* study of variability in ground reaction force [27] of walking speed range from 0.83m/s – 2.2m/s shows that except for the force component in the anteroposterior direction at normal walking speed (1.38m/s), the ground reaction force pattern varied with walking speed. Even more, there is variability in ground reaction force measurement with the same speed for each trial, which complicates the use of ground reaction force in the simulation. The careful inclusion of this aspect is being pursued in future work.

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