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DESIGN CONSIDERATIONS FOR AN ARTICULATED LEG-WHEEL LOCOMOTION SUBSYSTEM

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ABSTRACT

In this paper, we examine and evaluate candidate articulated leg-wheel subsystem designs for use in vehicle systems with enhanced uneven-terrain locomotion capabilities.

The leg-wheel subsystem designs under consideration consist of disk wheels attached to the chassis through an articulated linkage containing multiple lower-pair joints. Our emphasis is on creating a design that permits the greatest motion flexibility between the chassis and wheel while maintaining the smallest degree-of-freedom (d.o.f.) within the articulated chain. In particular, we focus our attention on achieving two goals: (i) obtaining adequate ground clearance by designing the desired/feasible motions of the wheel axle, relative to the chassis, using methods from kinematic synthesis; and (ii) reducing overall actuation requirements by a judicious mix of structural equilibration design and spring assist.

We examine this process in the context of two candidate designs – a coupled-serial-chain configuration and four-bar-configuration – for the articulated-leg-wheel subsystem. The performance of planar variants of these designs, operating in the sagittal plane, is evaluated and representative results are presented to highlight the process.

1. INTRODUCTION

In recent years, there has been considerable interest in creation of land-based locomotion systems for operation on rough unprepared surfaces. While high mobility, obstacle-surmounting capability and maneuverability are the obvious major requirements, additional criteria such as robustness, reliability and efficiency are extremely desirable features for systems operating on such unprepared terrains. Such systems would be extremely useful in a wide variety of application arenas such as exploration in extra-terrestrial [1]-[3], extreme terrestrial [4]-[6], and disaster environments [7].

Legged locomotion systems have been the preferred solution for maneuvering on such rough/unprepared terrains [8],[9] due to the ability to pick and choose the footholds and the natural suspension provided by the articulations. In contrast, wheeled systems have dominated on prepared surfaces [10]-[12] primarily due to the passive nature of the support and overall energy efficiency, making them the de-facto design standard for most man-made land-based locomotion systems. The limitation, however, is that these benefits are best realized on hard prepared surfaces, which can provide continuous support for wheels rolling without slip.

In this paper, we will examine creation of articulated leg-wheel subsystems for vehicle systems, as shown in Figure 1, to help providing enhanced locomotion capabilities on uneven terrain. In doing so, we wish to combine some of the benefits of legged and wheeled locomotion systems.

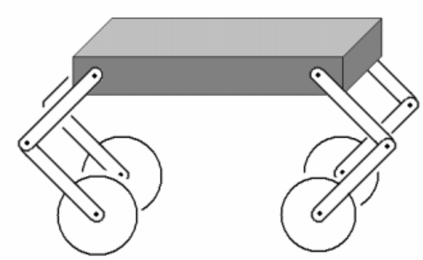


Figure 1: Artist's conception of an articulated leg-wheel based locomotion system.

The articulated leg-wheel subsystem designs under consideration consist of multiple lower pair joints (revolute/prismatic) between the wheel and the chassis. Numerous variants of the articulated leg-wheel system design are possible depending upon the type, number, sequencing and nature of actuation (active/passive) of the joint. Examples include the Mars Rover [1],[2] and SHRIMP with rocker bogie suspensions, the WAAV and NOMAD [4] with articulated frames to adapt to terrain roughness; to systems which possess powered legs and active/passive wheels like the Roller-Walker [7], WorkPartner [5],[6] and ALDURO [13]. In almost all these cases, it is the addition of articulations in the mechanical design that endows them with their superior locomotion capabilities. However, in almost all cases no systematic effort to "design" the articulated leg-wheel system is ever considered.

In general, any such design process must take into account innumerable, often equally-important, considerations such as the loss of stability, tip-over stability and ground traction for the task of locomotion on uneven terrain. However, in this paper, we will specifically focus our attention on two major complementary/ conflicting design criteria – workspace and suspension – in creating candidate designs for such articulated leg-wheel systems. A *large workspace* is desirable from the viewpoint of achieving adequate ground clearance and the ability to surmount obstacles and hence the open-kinematic structure of serial-chain based systems would be preferred. However, from the viewpoint of the *suspension,* typically we need an extremely small workspace and highly stiff articulations with minimal actuation requirements. In such situations, various types of closed-loop mechanical linkages have played an important role traditionally. A systematic selection between such competing criteria is one of the underlying themes in our work. We begin this process by considering two performance criteria – *articulated degrees-of-freedom* and *equilibration* – to help us characterize these many variants of the designs.

Articulated Degree-of-Freedom

In the sagittal plane, the wheel axle frame has two translation degrees-of-freedom (d.o.f.), relative to chassis, as shown in Figure 2 (since the orientation of the wheel rolling about its axis is irrelevant). However, a designer can control the number of d.o.f. that are retained/eliminated by changing types of articulations/joints (and the links) of the intermediate mechanism. This can range from: (i) eliminating all d.o.f. by introducing a rigid connection; (ii) retaining one d.o.f. by introducing a single lower-pair joint; and (iii) retaining the full two d.o.f. by introducing at least two lower-joint pairs as shown in Figure 2.

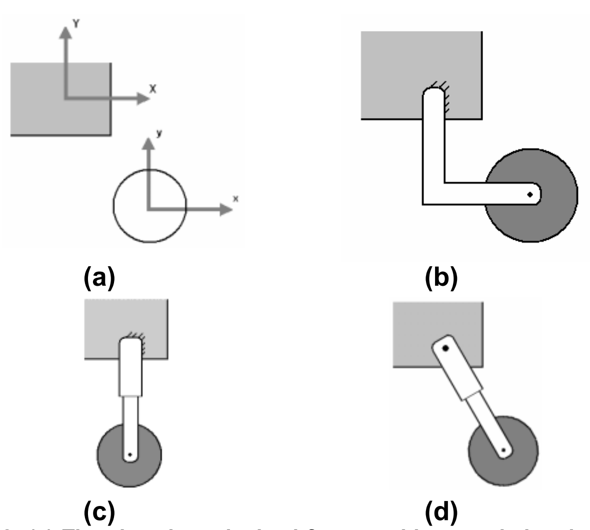


Figure 2: (a) The chassis and wheel frames with two relative d.o.f. and corresponding articulations with (b) zero d.o.f.; (c) one d.o.f.; and (d) two d.o.f.

Equilibration

Adding additional articulations increases the intermediate degree-of-freedom (d.o.f.) within the chain, which need to be controlled/restricted either: (i) *actively* by actuation, (ii) *semi-actively* using springs and dampers; or (iii) *passively* by adding some form of structural equilibration using hardware constraints. Since the load-bearing requirements can be significant, we will focus on the ability to use structural equilibration to bear these loads to the largest extent possible. The partitioning of the load bearing is highly configuration dependent and we will examine ways of customizing the configuration to enhance this process. Such a configuration selection by the designer can be a crucial design factor for designing a leg-wheel subsystem.

1.1 Single Degree-of-Freedom Articulated Leg-Wheel Systems

Current trends in robotics could enable us to increase the number of articulations (and actuation) within each subsystem in an attempt to create a "general-purpose solution" to overcome the increasing surface-roughness. However, we note that this increases the overall control complexity required even for basic operation of the vehicle on flat surfaces. Instead, in our work, we will make a case for creating *passive* articulated mechanical subsystems with built-in capability for graceful system degradation to a less capable (but stable) operational mode in cases of power-failure.

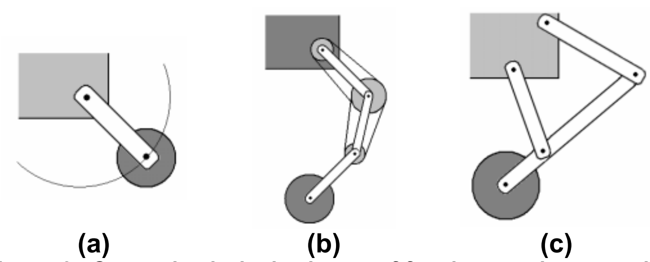


Figure 3: Constrained single degree-of-freedom motions can be achieved using: (a) a single lower-pair articulation; or multiple articulations but with suitable hardware constraints in (b) a Single Degree-of-Freedom Coupled Serial Chain (SDCSC) or (c) a four-bar configuration.

We focus our attention on various single degree-of-freedom (d.o.f.) articulated leg-wheel system designs – the simplest form is a single-link with a simple lower-pair joint. However, simple articulations such as these have very limited geometric motion capability – e.g. a revolute joint would limit the wheel-axle to move in a circle while a prismatic joint would constrain it to move in a straight line. Our emphasis in the selection process, however, will be on an articulated leg-wheel system design that permits the greatest motion capability between the chassis and wheel while maintaining the smallest d.o.f. within the articulated system. i.e. we would like to permit the wheel to trace a controlled complex geometric motion trajectory relative to the chassis while keeping the relative d.o.f. as small as possible.

In order to permit tracing of curves of greater complexity, we need to add more joints to increase geometric motion capability. However, this also increases the number of d.o.f. within the chain – constraints need to be added to the overall system in order to restrict the wheel-axle, which will form our end-effector, to a specific single d.o.f. desired path. We note that such constraints can be implemented either passively using hardware elements or actively by way of software control. In this paper, we will focus on two candidate designs which employ passive constraint implementation using hardware.

- The first candidate design employs the Coupled Serial Chain (CSC) configuration, as shown in Figure 3(b). Such a configuration is obtained by physically coupling the distal joint rotations of a revolute-jointed, multi-link, serial chain mechanism to the rotation of the proximal joint resulting in an Single Degree-of-freedom Coupled Serial Chain (SDCSC) mechanism. The rotation of the base link now drives all the other coupled links and increasing the number of links enables us to trace curves of increasing complexity and variety while retaining the single degree-of-freedom operation [15], [16].

- The alternate design is one that uses the well-known four-bar configuration as shown in Figure 3(c). The four-bar linkage is the simplest possible pin-jointed mechanism for single degree-of-freedom motion while allowing tracing of geometrically complex motion and force-profiles – this versatility is responsible for its ubiquitous use in machinery designs [18].

In this paper, we will emphasize the design-customization and adaptation of these two single d.o.f. designs for use on rough terrain to: (i) tackle increasing terrain roughness, and (ii) reduce actuation requirements.

The rest of the paper is organized as follows: Section 2 provides a brief background of the general kinetostatic design approach adopted. Sections 3 and 4 leverage this framework for kinetostatic design optimization for the cases of three-link SDCSC-based and fourbar-based leg-wheel subsystems, respectively. Finally, some concluding remarks and discussion are presented in Section 5.

2. DESIGN APPROACH

The general design problem will be formulated in the framework of dimensional synthesis of mechanisms [15]-[18].

2.1 Combined Precision-Point Synthesis & Optimization

In particular, given a set of task specifications and the type of mechanism, an optimization problem can be formulated to determine the set of parameters to match desired specifications. This process offers a systematic means for selection of large sets of unknown parameters. However, the resulting solutions satisfy all these desired specifications only in the least-squares sense without guaranteeing exact satisfaction of any specification.

Greater structure is added to this problem by employing Precision Point Synthesis (PPS). The requirement to match specifications exactly at precision points creates constraints between the various parameters of the mechanism, which aids the final selection of the parameters of the designed device. The design constraint equations themselves are created from the loop-closure equations for end-effector position specifications or by application of the Principle of Virtual Work for end-effector force specifications [15],[16]. These constraints can be combined within the previously described optimization-based framework to create a constrained design optimization problem.

Such a requirement for exact matching of more specifications at precision points creates more constraints. A limit on the number of specifications/points is finally reached and a simultaneous solution of the constraint system yields a unique mechanism. While exact matching at the greatest number of precision positions is desirable, the gains in terms of precise specification matching at many points may be offset by the disadvantage of having to solve a set of nonlinear equations.

Hence, in what follows, we consider: (i) the specification of less than maximum feasible number of precision points; (ii) a suitable partitioning of all the variables into dependent and independent variables; (iii) a suitable specification of the independent variables; and (iv) leading up to the formation of a linear system of equations in terms of the dependent variables. The optimization over the independent variables yields different candidate mechanisms, which satisfies the design specifications exactly at the selected precision points and in the least-squares sense elsewhere. The principal advantage of this approach is the ability to add structure to the problem while offering adequate flexibility/choices to the designer. We also note that while we focus on matching the desired specifications on end-effector positions and forces, other types of design specifications can also be explored.

2.2 Kinetostatic Design of the Leg-Wheel Subsystem

The design of an articulated leg-wheel system that is capable of surmounting an obstacle using minimal actuation forces while supporting the weight of the chassis offers many challenges. Suitable selection of various mechanism configurations and mechanism design parameters is critical and includes both *kinematic* parameters, such as the link-lengths and initial configuration, as well as *static* parameters such as spring constants and their preloads.

First, we formulate the problem as a *kinematic path-following problem*, i.e. a problem of selecting the parameters to permit the wheel axle (which serves as the end-effector) to follow a desired pre-specified path relative to the chassis (which serves as the base). However, the force interaction between a linkage and its environment also becomes critical to the performance (as in the case of our leg-wheel design). In this case, the goal of the articulation is to guide the attached wheel axle through several positions while supporting *a set of specified external loads.* Building on the work presented in [15],[16], we examine the kinetostatic design process for selection of the optimal configuration (and determine the minimum torques required) to support these external loads by *structural equilibration*. We also consider the addition of torsional springs at each joint to minimize/optimize the peak actuator torques by a judicious combination of *structural equilibration design* and *spring assists*.

In general, a simultaneous determination of optimal mechanism and spring parameters is possible by formulating and simultaneously solving the loop-closure equations and static equilibrium equations at the precision points. However, for the discussion in the next two sections, we will assume that a *kinematic path-following* optimization scheme has been previously performed and the results are available. This enables us to focus solely on the creation and satisfaction of the *static precision-point constraints*. In view of the limited space, we will present the principal formulation details and results for the kinetostatic design case due to its obvious relevancy (and also because it subsumes the kinematic design as a special subcase). The interested reader is referred to [19] for further details.

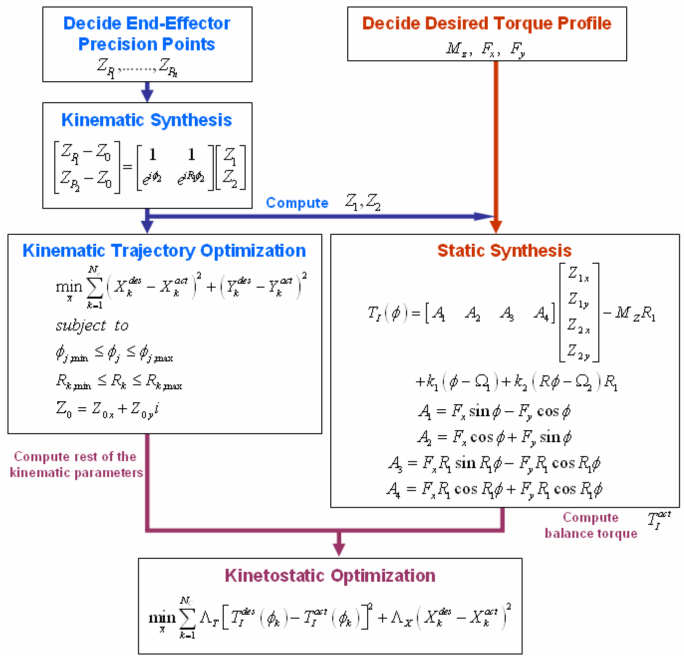


Figure 4: Design approach combining Kinematic and Kinetostatic Precision Point Synthesis with Optimization.

2.3 Specific Test Scenario

In this paper, we will consider an ideal profile of step that was determined using the UBC (Uniform Building Codes) to have a tread of 7 1/2” and a rise of 9 1/2". Further, the specification of a desired kinematic motion curve would typically consist of specification of the motion of the axle with respect to a coordinate frame attached to the chassis of the moving vehicle. Thus, we note that the creation of the desired relative motion trajectory depend critically upon selection of a chassis velocity. We will nominally assume a fixed chassis for the rest of this paper. Finally, a designer has freedom in selecting the type, number and locations of the precision points at which an exact matching of the desired motion trajectories would be desired. We select these to be the start, the corner and the end of the step as seen in Figure 5(a).

The “desired torque curve” is a plot of the torque (required to equilibrate a vertical end-effector load of 2 N) vs. the relative motions of the mechanism from its initial configuration, as seen in Figure 5(b). We will consider a “hill-shaped” torque curve with two nominal equilibrium/rest positions at either end of the motion range. Thus in each of these rest configurations, we would like to equilibrate the applied loads without requiring any actuation. Further, when the wheel axle is displaced from its initial configuration we would like the system torque to move the system back to the initial configuration. However, when a certain threshold value of torque applied is exceeded, we require the system torques to move the system to the other equilibrium position which is selected to be the other end of the range of motion.

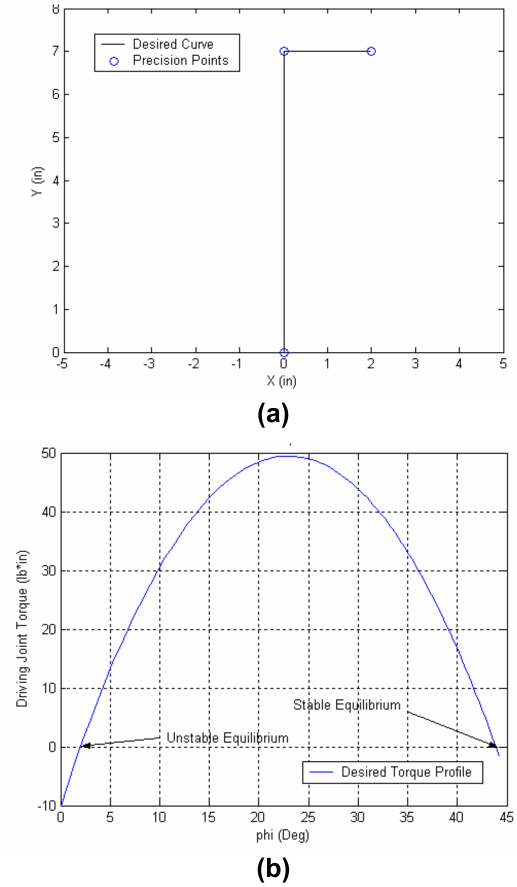


Figure 5: (a) Desired kinematic curve and (b) desired torque curve.

3. DESIGN/SYNTHESIS OF SDCSC LEG-WHEEL MECHANISM

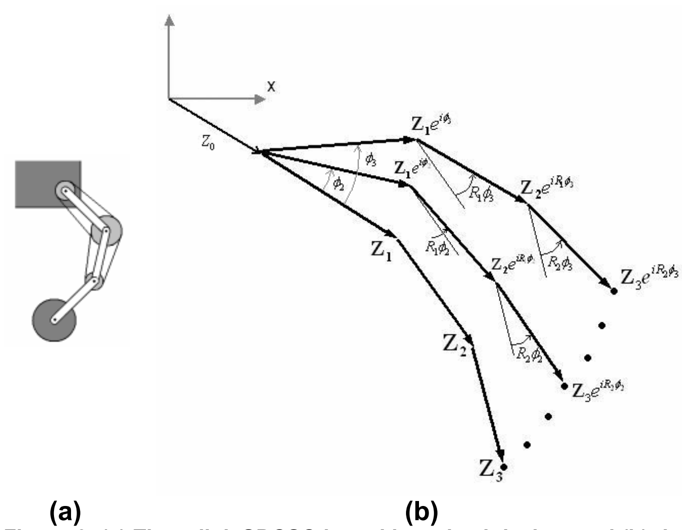


Figure 6: (a) Three-link SDCSC-based leg-wheel design, and (b) the corresponding kinematic diagram.

3.1 Kinematic Synthesis

We set the desired trajectory of the wheel-axle to be the standard step described in Section 2.3. The overall kinematic design selection problem may be expressed in the form of a constrained optimization problem as:

$$\min_{\phi_2,\phi_3,R_1,R_2} \sum_{k=1}^{N_c}\left[\left(P_{xk}-Q_{xk}\right)^2+\left(P_{yk}-Q_{yk}\right)^2\right] \quad (1)$$

subject to:

$$\phi_{\min} \le \phi_2 \le \phi_{\max}\,,\ \phi_{\min} \le \phi_3 \le \phi_{\max}\,,\ \phi_{\min} > 0$$

$$R_{1,\min} \le R_1 \le R_{1,\max}\,,\ R_{2,\min} \le R_2 \le R_{2,\max}$$

$$\begin{bmatrix}\mathbf{Z}_1\\ \mathbf{Z}_2\\ \mathbf{Z}_3\end{bmatrix} = \begin{bmatrix}1 & 1 & 1\\ e^{i\phi_2} & e^{iR_1\phi_2} & e^{iR_2\phi_2}\\ e^{i\phi_3} & e^{iR_1\phi_3} & e^{iR_2\phi_3}\end{bmatrix}^{-1}\begin{bmatrix}\mathbf{Z}_{P_1}-\mathbf{Z}_0\\ \mathbf{Z}_{P_2}-\mathbf{Z}_0\\ \mathbf{Z}_{P_3}-\mathbf{Z}_0\end{bmatrix} \quad (2)$$

where $\mathbf{Q}_k = Q_{x,k}+iQ_{y,k}$ and $\mathbf{P}_k = P_{x,k}+iP_{y,k}$ correspond to the k^{th} desired and actual position of the end-effector at N_c discrete correspondence points along the curve. The design variables correspond to base rotation angles ϕ_2 and ϕ_3 at the two precision points and the two pulley ratios R_1 and R_2. These are further constrained to match the requirements imposed by physical realization. Furthermore, the position-vector of the base joint, $\mathbf{Z}_0 = Z_{0x}+iZ_{0y}$ is fixed with respect to the chassis, and length of the first link are also limited to satisfy the feasibility constraints. The constraint equations corresponding to the kinematic precision point synthesis are derived from the forward kinematics equations evaluated at each precision point.

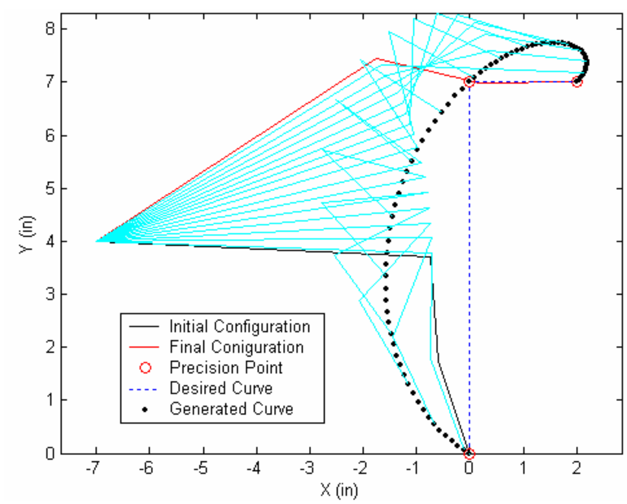


Figure 7: Candidate SDCSC configuration with design variables of $R_1=-8$, $R_2=2$, $\phi_2=20.9°$, $\phi_3=36°$.

Figure 7 shows the various kinematic configurations of the mechanism that approximates the step closely – in bold at the three precision points and in lighter color along the rest of the curve.

3.2 Kinetostatic Synthesis

We note that the moments and forces at the end-effector are intimately dependent upon the slope of the ground, but more importantly are dependent upon (and can be controlled by) the motor torques applied on the wheel at the axle. Figure 8 depicts the free-body diagram of the wheel-axle and wheel. We assume that the weight of chassis creates a vertical force F_g at the axle and the motor mounted at the axle exerts a torque of T_{motor} on the wheel. Assuming a ground slope of α, we can compute the resultant axle forces F_x F_y and moments M_z can be computed as:

$$\begin{aligned} F_x &= F_{ik,x} = -\frac{T_{motor}}{R}\cos\alpha \\ F_y &= F_{ik,y} = -\frac{T_{motor}}{R}\sin\alpha + F_g \\ M_z &= M_{ik} = -T_{motor} + F_g R\sin\alpha \end{aligned} \quad (3)$$

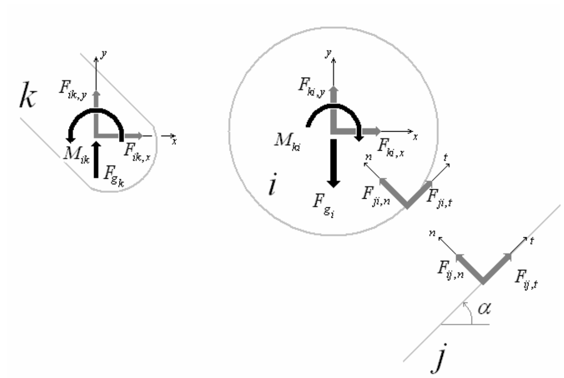


Figure 8: Free-body diagram of wheel axle supporting an external force and torque

The constraints on the static torque are derived from the Principle of Virtual Work for every precision position. The actual torque, $T_I(\phi)$, applied at the base joint can be divided into two parts: (i) the torque required to equilibrate the external load, T_{Ext}, and (ii) the contribution of the internal spring torques, T_{Spr}, that can help reduce the overall required equilibrating torque on the base joint, and $T_I = T_{Ext} - T_{Spr}$. The torque required to equilibrate the external load, T_{Ext}, can be expressed as:

$$T_{Ext} = \begin{bmatrix}A_1 & A_2 & A_3 & A_4 & A_5 & A_6\end{bmatrix}\begin{bmatrix}z_{1x}\\ z_{1y}\\ z_{2x}\\ z_{2y}\\ z_{3x}\\ z_{3y}\end{bmatrix} + M_z R_2$$

$$\begin{aligned} A_1 &= -F_x\sin\phi + F_y\cos\phi \\ A_2 &= -F_x\cos\phi - F_y\sin\phi \\ A_3 &= -F_xR_1\sin\left(R_1\phi\right) + F_yR_1\cos\left(R_1\phi\right) \\ A_4 &= -F_xR_1\cos\left(R_1\phi\right) - F_yR_1\sin\left(R_1\phi\right) \\ A_5 &= -F_xR_2\sin\left(R_2\phi\right) + F_yR_2\cos\left(R_2\phi\right) \\ A_6 &= -F_xR_2\cos\left(R_2\phi\right) - F_yR_2\sin\left(R_2\phi\right) \end{aligned} \quad (4)$$

In order to calculate the spring torques, we introduce the following notation. Let θ_i be the absolute initial configuration of each joint, Ω_i be the absolute configuration of each joint at which the springs are assembled, and ϕ be the relative displacement of the first link from its initial configuration to the current configuration. Then, β_i, the angular extension of each spring, can be written as:

$$\beta_1 = \theta_1 + \phi - \Omega_1$$
$$\beta_i = \left[\left(\theta_i + R_{i-1}\phi - \Omega_i \right) - \left(\theta_{i-1} + \phi - \Omega_{i-1} \right) \right], \ \forall i > 2 \quad (5)$$

Thus, the overall contribution of the internal spring torques can be expressed as:

$$\begin{aligned} T_{Spr} &= k_1 \left(\theta_1 + \phi - \Omega_1 \right) \\ &+ k_2 \left[\left(R_1 - 1 \right)\phi + \left(\theta_2 - \theta_1 \right) + \left(\Omega_1 - \Omega_2 \right) \right]\left(R_1 - 1 \right) \\ &+ k_3 \left[\left(R_2 - R_1 + 1 \right)\phi + \left(\theta_3 - \theta_2 + \theta_1 \right) + \left(\Omega_2 - \Omega_3 - \Omega_1 \right) \right]\left(R_2 - R_1 + 1 \right) \\ &= \left[k_1 + k_2 \left(R_1 - 1 \right)^2 + k_3 \left(R_2 - R_1 + 1 \right)^2 \right]\phi \\ &+ k_1 \left(\theta_1 - \Omega_1 \right) + k_2 \left(R_1 - 1 \right)\left[\left(\theta_2 - \theta_1 \right) + \left(\Omega_1 - \Omega_2 \right) \right] \\ &+ k_3 \left(R_2 - R_1 + 1 \right)\left[\left(\theta_3 - \theta_2 + \theta_1 \right) + \left(\Omega_2 - \Omega_3 - \Omega_1 \right) \right] \end{aligned} \quad (6)$$

Nominally, a system with three springs requires the specification of 6 variables – k_1, k_2, k_3, Ω_1, Ω_2, Ω_3. However, we note that the spring torque equation can also be rewritten in terms of the relative angular motion of the first joint as a linear equation as:

$$T_{Spr}\left(\phi \right) = a\phi + b = a\left[\theta_1 + \phi + \left(\frac{b - a\theta_1}{a} \right) \right] \quad (7)$$

with the implication that all linear torsional springs (attached at the various joints of an SDCSC mechanism) can in fact be replaced by a single equivalent torsional spring with spring constant $K_{eq} = a$ and an initial unloaded configuration of $\Omega_{eq} = -\dfrac{b - a\theta_1}{a}$. The spring constants k_1, k_2, k_3 are related to a and b through the following underdetermined system:

$$\begin{bmatrix} a \\ b \end{bmatrix} = \left[S \right] \begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix} = \begin{bmatrix} -1 & -\left(R_1 - 1 \right)^2 & -\left(R_2 - R_1 + 1 \right)^2 \\ s_{21} & s_{22} & s_{23} \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix}$$
$$s_{21} = -\left(\theta_1 - \Omega_1 \right) \quad (8)$$
$$s_{22} = -\left(R - 1 \right)\left[\left(\theta_2 - \theta_1 \right) + \left(\Omega_1 - \Omega_2 \right) \right]$$
$$s_{23} = -\left(R_2 - R_1 + 1 \right)\left[\left(\theta_3 - \theta_2 + \theta_1 \right) + \left(\Omega_2 - \Omega_3 - \Omega_1 \right) \right]$$

Since there are more unknowns than equations we note that a secondary optimization in terms of the k_i's may be easily created. We note that a naive (but direct) solution for the unknown spring constants using pseudo-inverse of $\left[S \right]$ matrix may give rise to negative k_i's. Such infeasible values for k_i's can be avoided by adding components from the nullspace to ensure positive values for all k_i's. Regardless, it is reasonably evident that the designer has considerable freedom to either (i) determine the best unloaded configuration for a given k_1, k_2, and k_3; or alternately (ii) vary the initial unloaded configuration to enable suitable selection of a spring constant. The interested reader is referred to [19] for further details.

The desired torque curve is assumed to be given as a function of ϕ as discussed in Section 2.3. However, from Eq. (7) we note that we only have two design variables for use in the static synthesis process (regardless of number of links of SDCSC mechanism) which can be used in the following ways:

- Two static precision points and no free choices (presented below in Section 3.2.1);
- One static precision point and using one of the variables as a free choice (not presented);
- No static precision points but with both variables as free-choices (in Section 3.2.2)

3.2.1 Two static precision points and no free choices

For a two static precision position problem, the spring constants are calculated by substituting precision torques – the specified torques, $T_I\left(\phi_i \right)$ at the precision points, ϕ_i in Eq. (7).

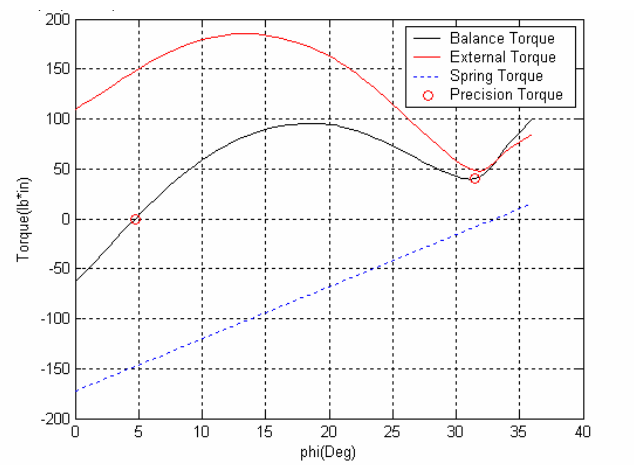


Figure 9: Torque at driving joint of three-link SDCSC configuration with two precision torques.

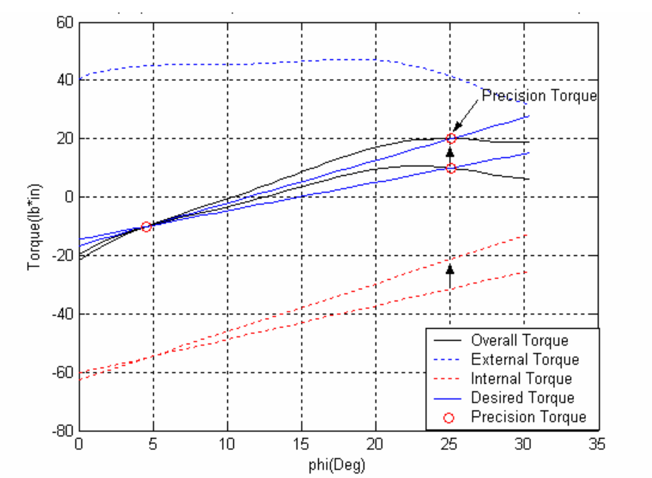


Figure 10: Altering the torque profile by changing the precision torque values.

Figure 9 shows the corresponding torque profile – note that a zero first precision torque was included to demonstrate the capability of designing the rest/equilibrium position. Using two precision points in static PPS eliminates all freedom and we get a unique torque profile for any kinematic configuration. Thus, we can move the torque profile only by changing the precision torque condition, as shown in Figure 10.

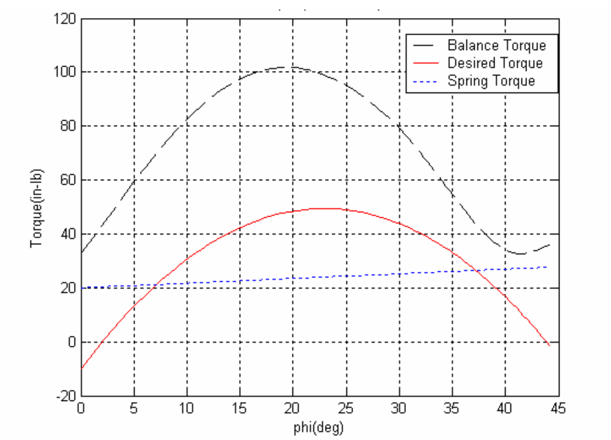
3.2.2 No static precision points and using both variables as free-choices

A variety of objective functions including force consideration in their objective can be employed as noted in [15][16]. The most satisfying solution, however, is obtained by minimizing the peak input torque and the discrepancy between desired and generated curves of end-effector over the entire range of motion. The optimization is carried out over the many candidate solutions to obtain the best specification. The torque at Joint 1, T_I, is written as:

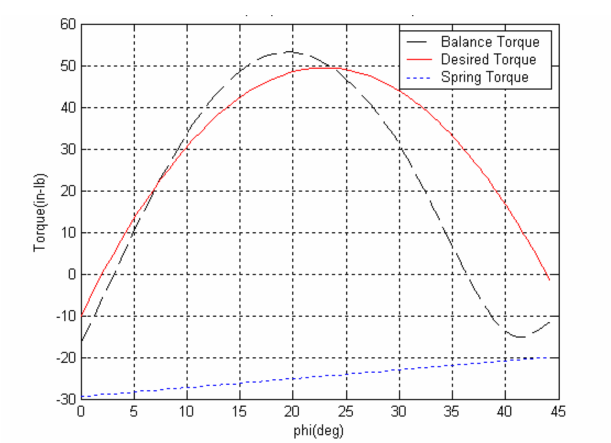
$$T_I = T_{Ext} - T_{Spr} = T_{Ext} + \left(a\phi + b\right) \tag{9}$$

and using a and b as the design variables, the optimization problem can be stated as:

$$\min_{a,b} \sum_{k=1}^{N_c} \left(T_I^{des} - T_I^{act}\right)^2 \tag{10}$$



(a)



(b)

Figure 11: Desired torque optimization for a three-link SDCSC configuration (a) after first iteration of optimization, and (b) final result of optimization.

Figure 11 shows the result of such an optimization in terms of the torque profiles corresponding to the kinematic configurations represented in the Section 2. Figure 11 (a) depicts the first iteration of optimization, and Figure 11(b) shows the final optimized torque profile. In the SDCSC static optimization, the approximation of the generated torque to the desired curve depends mainly on the external torque (which determined by the kinematic configuration and the external loads). Because the internal torque of an SDCSC-based design is a linear function, we cannot match complex profiles with rapid changes of torque.

4. DESIGN/SYNTHESIS OF FOUR-BAR LEG-WHEEL MECHANISM

The four-bar linkage has found ubiquitous applicability in machinery applications due to its extreme versatility in modulating transmitted motions and forces [18]. We will similarly exploit this versatility in designing an articulated leg-wheel system based on the four-bar configuration.

4.1 Kinematic Synthesis

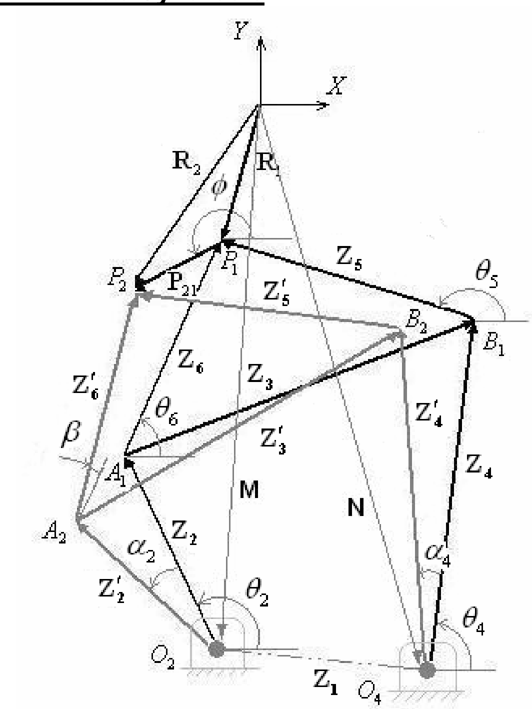


Figure 12: Nomenclature of the four-bar linkage at two precision positions.

A dyadic synthesis approach outlined in greater detail in Norton [18] is adopted for generating the kinematic constraint equations. As in the case of the SDCSC mechanism, the number of feasible precision points is limited. The four-bar linkage can be synthesized by closed-form methods up to five precision points for the path-following problem. The four or more precision point synthesis problems involve the solutions of nonlinear equations – hence we consider only two or three precision points cases. We set desired trajectory of the wheel-axle to be the standard step described in Section 2. The overall kinematic design selection problem may be expressed in the form of a constrained optimization problem as:

$$\min_{L_2,L_3,\alpha_2,\alpha_4,\beta} \sum_{k=1}^{N} \left[\left(P_{xk} - Q_{xk}\right)^2 + \left(P_{yk} - Q_{yk}\right)^2\right] \tag{11}$$

subject to

$$\begin{bmatrix} \mathbf{Z}_2 \\ \mathbf{Z}_6 \\ \mathbf{Z}_4 \\ \mathbf{Z}_5 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ e^{i\alpha_2} & e^{i\beta} & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & e^{i\alpha_4} & e^{i\beta} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{R}_1 - \mathbf{M} \\ \mathbf{R}_2 - \mathbf{M} \\ \mathbf{R}_1 - \mathbf{N} \\ \mathbf{R}_2 - \mathbf{N} \end{bmatrix}$$

$$L_{2,\min} \le L_2 \le L_{2,\max},\ L_{3,\min} \le L_3 \le L_{3,\max},$$
$$\alpha_{2,\min} \le \alpha_2 \le \alpha_{2,\max},\ \alpha_{4,\min} \le \alpha_4 \le \alpha_{4,\max},$$
$$\beta_{\min} \le \beta \le \beta_{\max}$$

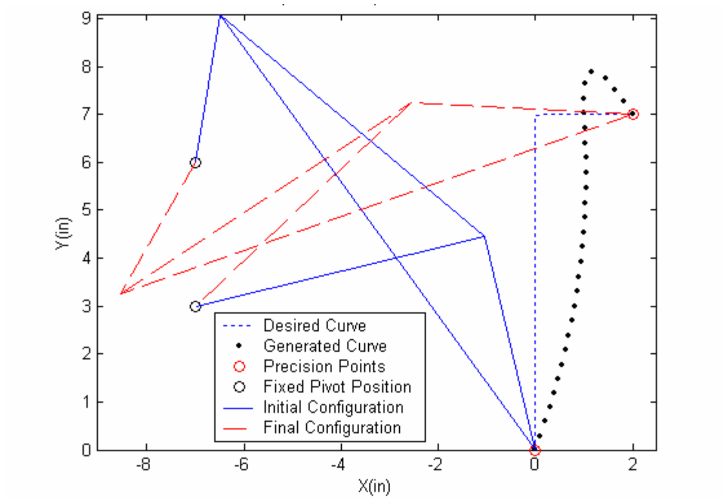


Figure 13: Kinematic configuration of four-bar for desired step trajectory.

M and **N** are constant vectors and the design variables are the precision point angles $\alpha_2, \alpha_4, \beta$, and the second/third link-lengths L_2, L_3. $\mathbf{P}_k = P_{x,k} + iP_{y,k}$ is the generated end-effector position and $\mathbf{Q}_k = Q_{x,k} + iQ_{y,k}$ is desired end-effector position. Among the possible configurations, an acceptable configuration of four-bar is shown in Figure 13.

4.2 Kinetostatic Synthesis

As in the case of the SDCSC-based design, the expression for the torque due to external sources at the driving joint T_{Ext} and the spring torque, T_{Spr} can be calculated from the expression for virtual work in the system [19] as:

$$\begin{aligned} T_{Ext} &= -F_x\left[z_2 \sin\theta_2 + A_1 z_6 \sin(\theta_3 + \gamma_6)\right] \\ &\quad + F_y\left[z_2 \cos\theta_2 + A_1 z_6 \cos(\theta_3 + \gamma_6)\right] + A_1\mathbf{M} \\ T_{Spr} &= -k_2(\theta_{2,initial} + \alpha_2 - \Omega_2) \\ &\quad - k_3(A_1 - 1)\left[(\theta_{3,initial} + \beta - \Omega_3) - (\theta_{2,initial} + \alpha_2 - \Omega_2)\right] \\ &\quad - k_4 A_2(\theta_{4,initial} + \alpha_4 - \Omega_4) \\ &\quad - k_5(A_1 - A_2)\left[(\theta_{5,initial} + \beta - \Omega_5) - (\theta_{4,initial} + \alpha_4 - \Omega_4)\right] \end{aligned} \quad (12)$$

This spring equation can also be rewritten in terms of the relative angular motion of the three joints as a linear equation.

$$T_{Spr} = a\alpha_2 + b\alpha_4 + c\beta + d \quad (13)$$

However, in a four-bar mechanism it is possible to express α_4 and β as nonlinear functions of the independent joint angle α_2. These relationships can be written as $\alpha_4(\alpha_2)$ and $\beta(\alpha_2)$ and are determined by the selected kinematic configuration. Since the total required actuation is the sum $T_{Ext} + T_{Spr}$, which depends both on the selected configuration as well as the spring parameters. As in the previous section, we adopt a decoupled approach in which the kinematic configuration is predetermined in a kinematic design-optimization stage. Thus from Eq. (13), we only have four design variables (a, b, c, d) for use in the static synthesis process of a four-bar mechanism. While this allows examination of many cases, we will examine the following two cases:

- Four static precision points and no free choices (in Section 4.2.1).
- Without static precision points and using four variables as free-choices (in Section 4.2.2).

4.2.1 Four static precision points and no free choices

For a four static precision position problem, the spring constants and spring preloads are calculated by substituting precision torques – the specified torques, $T_i(\phi_i)$, at the precision points, α_2 in Eq. (13).

$$\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} \alpha_{2,1} & \alpha_4(\alpha_{2,1}) & \beta(\alpha_{2,1}) & 1 \\ \alpha_{2,2} & \alpha_4(\alpha_{2,2}) & \beta(\alpha_{2,2}) & 1 \\ \alpha_{2,3} & \alpha_4(\alpha_{2,3}) & \beta(\alpha_{2,3}) & 1 \\ \alpha_{2,4} & \alpha_4(\alpha_{2,4}) & \beta(\alpha_{2,4}) & 1 \end{bmatrix}^{-1} \begin{bmatrix} T_{Joint,1} - T_{Ext}(\alpha_{2,1}) \\ T_{Joint,2} - T_{Ext}(\alpha_{2,2}) \\ T_{Joint,3} - T_{Ext}(\alpha_{2,3}) \\ T_{Joint,4} - T_{Ext}(\alpha_{2,4}) \end{bmatrix} \quad (14)$$

The desired k_i and corresponding Ω_i can be decided by solving:

$$\begin{bmatrix} k_2 \\ k_3 \\ k_4 \\ k_5 \end{bmatrix} = \begin{bmatrix} -1 & (A_1 - 1) & 0 & 0 \\ 0 & 0 & -A_2 & (A_1 - A_2) \\ 0 & -(A_1 - 1) & 0 & -(A_1 - A_2) \\ m_{41} & m_{42} & m_{43} & m_{44} \end{bmatrix}^{-1} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$
$$\begin{aligned} m_{41} &= -(\theta_{2,initial} - \Omega_2) \\ m_{42} &= -(A_1 - 1)\left[(\theta_{3,initial} - \Omega_3) - (\theta_{2,initial} - \Omega_2)\right] \\ m_{43} &= -A_2(\theta_{4,initial} - \Omega_4) \\ m_{44} &= -(A_1 - A_2)\left[(\theta_{5,initial} - \Omega_5) - (\theta_{4,initial} - \Omega_4)\right] \end{aligned} \quad (15)$$

Note that as in the SDCSC-based design, Eq. (15) gives us freedom for selection of spring constants. For example, noting that commercially available springs have restricted values of spring constants, a secondary optimization to minimize them can be easily performed using spring preloads as design variables.

In Figure 14, we set the first precision torque as negative to increase the system's stiffness to the disturbance and determine second torque as zero to demonstrate the capability of designing stable/unstable equilibrium position. To make it easy to move the system's configuration to the alternate configuration, we set third torque to be a positive value. Note that this obtained torque-profile corresponds to the previously determined kinematic trajectory of Figure 13. Changing the kinematic trajectory will alter the torque profile and can serve as an additional useful source of freedom for the designer.

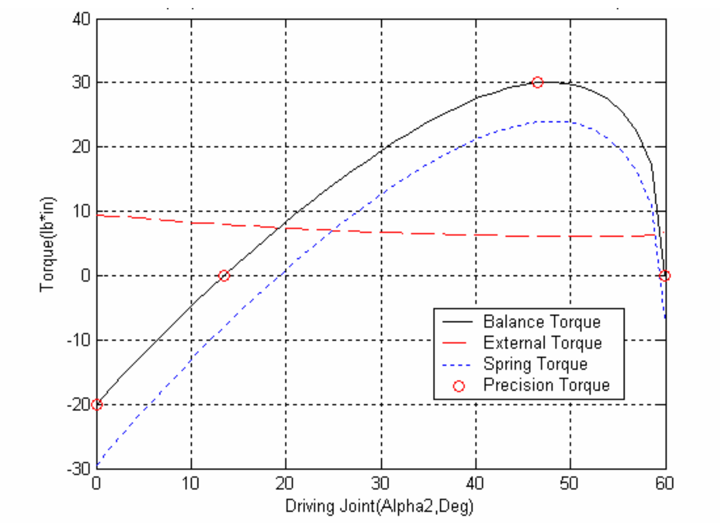


Figure 14: Driving torque in the four-bar configuration.

4.2.2 No static precision points and using four variables as free-choices

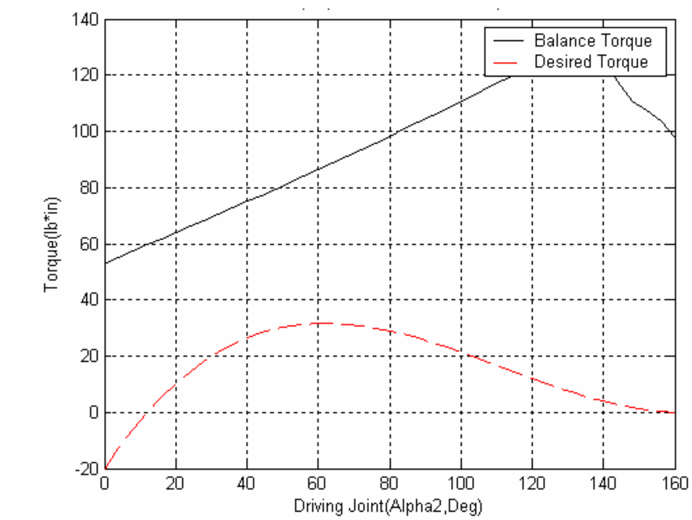
As in the case of SDCSC, we minimize the discrepancy between the desired and a generated curve of the end-effector over the entire range of motion. Optimization is carried out over the many candidate solutions to obtain the best specification. Now we introduce possible sets of minimizations including force consideration in their objective and using a, b, c, d in Eq. (13) as the design variables.

Figure 15 shows the result of such an optimization in terms of the torque profiles corresponding to the kinematic configuration in Figure 13. Figure 15 (a) depicts the first iteration of the optimization and Figure 15 (b) shows the final optimized torque profile. In contrast to the torque optimization with the SDCSC-based design seen in Figure 11, we see that a four-bar based design is better able to match complex torque profiles.

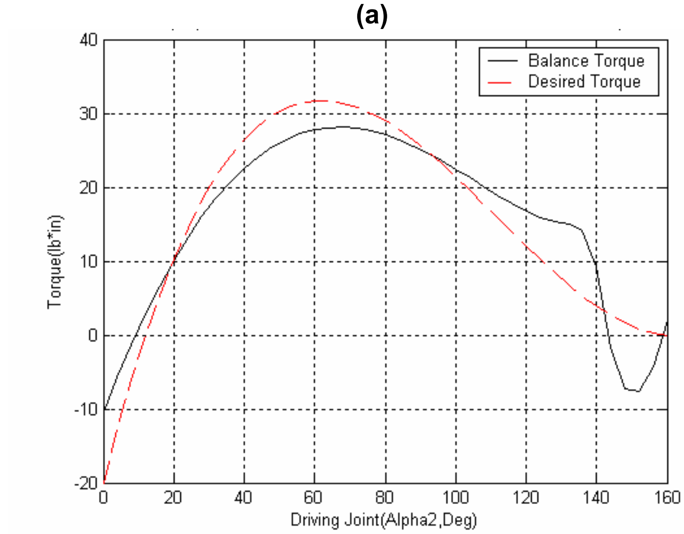
5. CONCLUSION

In this paper, we considered the performance evaluation of alternate designs of articulated leg-wheel subsystems to modulate the motion and force interactions between the ground and the chassis and thereby enable vehicle systems to locomote in difficult environments and rough terrain.

We focused our attention on passive mechanical implementation of such single degree-of-freedom intermediate articulation using two configurations: (i) a four-bar-based design and (ii) an SDCSC-based design. We emphasized the design-customization and adaptation of these two single degree-of-freedom mechanisms for use on rough terrain. We examined the application of kinematic and kinetostatic design optimization methods to select design parameters of the leg-wheel subsystems to allow them to tackle increasing terrain roughness, achieve better chassis-terrain decoupling and reduce actuation requirements. We also examined the influence of the mechanical parameters on the static equilibrium curves of the articulated leg-wheel subsystem from the viewpoint of developing future capabilities to customize/tailor these mechanical system responses to desired behaviors.



(a)



(b)

Figure 15: Desired torque optimization for a four-bar (a) at first iteration and (b) final optimization result.

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REFERENCES

[1] Hacot, H., Dubowsky, S., and Bidaud, P., 1998, "Modeling and Analysis of a Rocker-Bogie Planetary Exploration Rover," Proceedings of the Twelveth CISM-IFToMM Symposium (RoManSy 98), Paris, France.

[2] Hacot, H., 1998, "Analysis and Traction Control of a Rocker-Bogie Planetary Rover," M.S. Thesis, Department of Mechanical Engineering, MIT, Cambridge, MA, USA.
[3] Merminod, B., Lauria, M., Piguet, R., and Siegwart, R., 2000, "An Innovative Space Rover with Extended Climbing Abilities," Proceedings of the Space and Robotics 2000, Albuquerque, NM, USA.
[4] Wettergreen, D., Bualat, M., Christian, D., Schwehr, K., Thomas, H., Tucker, D., and Zbinden, E., 1997, "Operating Nomad during the Atacama Desert Trek", Proceedings of Field and Service Robotics Conference, Canberra, Australia.
[5] Halme, A., Leppänen, I., and Salmi, S., 1999, "Development of Workpartner-Robot – Design of Actuating and Motion Control System," CLAWAR'99, Portsmouth, UK.
[6] Halme, A., Leppänen, I., Montonen, M., and Ylönen, S., 2001, "Robot Motion by Simultaneous Wheel and Leg Propulsion," 4th International Conference on Climbing and Walking Robots, Karlsruhe, Germany, Professional Engineering Publishing Ltd, pp. 1013-1020.
[7] Hirose, S., 2000, "Super Mechano-System," International Symposium on Experimental Robotics (ISER2000), Honolulu, HI, USA.
[8] Song, S-M. and Waldron, K., 1988, *The Adaptive Suspension Vehicle*, MIT Press, Cambridge, MA, USA.
[9] Berns, K., 2002, *The Walking Machine Catalogue* URL:http://www.walking-machines.org/.
[10] Bekker, M., 1956, *Theory of Land Locomotion: The Mechanics of Vehicle Locomotion: The mechanics of Vehicle Mobility*, The University of Michigan Press, Ann Arbor, MI, USA.
[11] Bekker, M., 1969, *Introduction to Terrain Vehicle Systems,* The University of Michigan Press, Ann Arbor, MI, USA.
[12] Wong, J., 1989, *Terramechanics and Off-road Vehicles*, Elsevier, Amsterdam, Netherlands.
[13] Hiller, M., and German, D., 2002, "Manoeuvrability of the Legged and Wheeled Vehicle ALDURO in Uneven Terrain with Consideration of Nonholonomic Constraints" Proceeding of ISOM 2002, Chemnitz, Germany.
[14] Iagnemma, K., Rzepniewskia, A., Dubowsky, S., Pirjanian, P., Huntsberger, T., and Schenker, P., 2000, "Mobile Robot Kinematic Reconfigurability for Rough-Terrain," in Proceedings SPIE's International Symposium on Intelligent Systems and Advanced Manufacturing, Boston, MA, USA.
[15] Krovi, V., 1998, "Design and Virtual Prototyping of User-Customized Assistive Devices," Ph.D. Thesis, Dept. Of Mechanical Engineering and Applied Mechanics, University of Pennsylvania, Philadelphia, PA, USA.
[16] Krovi, V., Ananthasuresh G.K., and Kumar, V., 2002, "Kinematic and Kinetostatic Synthesis of Planar Coupled Serial Chains" ASME Journal of Mechanical Design, **124(**2), pp.143-155.
[17] Sandor, G.,and Erdman, A.G., 1984, *Advanced Mechanism Design: Analysis and Synthesis,* Vol. 2. Prentice Hall International, Englewood Cliffs, NJ, USA.
[18] Norton, R., 2004, "Design of Machinery," 3rd Edition, McGraw-Hill, New York, NY, USA.
[19] Jun, S. K., 2004, "*Kinetostatic Design of an Articulated Leg-Wheel Locomotion Subsystem*," M.S. Thesis, Department of Mechanical and Aerospace Engineering, University at Buffalo, Buffalo, NY, USA.